Ghost- Free Higher Derivative Quantum Gravity, the Hierarchy and the Cosmological Constant Problems

V. I. Tkach
Department of Physics and Astronomy,
Northwestern University, Evanston, IL 60208-3112, USA
v-tkach@northwestern.edu

June 3, 2009

Abstract

Proposing a new solution to problems of the hierarchy and smallness of the cosmological constant using the Tev scale of the Standard Model in a new framework of the higher-order gravity.

PACS number: 04.20.Fy; 04.60.Ds; 12.60.Jv; 98.70.Dk.
Keywords: gravity, hierarchy problem, dark energy

The theory of higher derivative gravitation, whose action contains terms quadratic in the curvature in addition to the Einstein term, is a renormalizable field theory [1−7], but it is not free defect. This theory gives rise to unphysical poles in spin-two sector of the tree- level propagator which break the unitarity.

On the other hand, induced gravity program with fourth –order gravitational theories [8−12] that does not contain dimensional coupling constants and the unphysical ghosts, but in such theories Newton's constant is not calculable and is a free parameter [13].

In this note we present an example of the quantum gravity with the higher curvature which is ghost-free.

We show that the gravitational strength and other observed fundamental interactions are the consequence of one fundamental dimension scale about 10^3 Gev , which is vacuum expectation values the Higgs fields $v{=}M_{\varepsilon w} \approx 10^3 Gev$ of the Standard Model. This is a new solution to the well-known hierarchy problem in physics.

We proposed also a solution of the vacuum energy density which depends of the Tev scale and the coupling constants higher order curvature terms (in the frame new R^2-gravity).

Throughout, units have been chosen such that $c = \hbar = 1$.

Let us start with the action

$$S = \int d^4x\sqrt{-g}[-\frac{\epsilon}{2}\left(v^2 - \Phi^+\Phi\right)R + aW - \frac{b}{3}R^2 - \tag{1}$$

$$-\frac{1}{2}\left(D_\mu\Phi\right)^+\left(D^\mu\Phi\right) - f\left(\Phi^+\Phi - v^2\right)^2] + S_{sm},$$

where S_{sm} is a part of the action Standard Model for gauge fields and the fermion fields.

Spin zero doublet [14] the Higgs field is Φ, the Weyl term W is $W = R^2_{\mu\rho} - \frac{1}{3}R^2$, where $R_{\mu\rho}$ is the Ricci tensor and R is the scalar curvature. In the action (1) ϵ, a, b and f are dimensionless coupling constants.

The term in the action (1) $-\frac{1}{2}\epsilon v^2 R\sqrt{-g}$ is the Einstein term, where we have rule $\epsilon v^2 = M_p^2$ and $M_p = (8\pi G)^{-\frac{1}{2}} \sim 10^{18} Gev$ is the reduced Planck mass, so the Newton constant G is not a fundamental constant.

The higher-curvature terms in the action (1) will have little effect at low energies compared to the Einstein term. At the lowest energy, only $-\frac{1}{2}\epsilon v^2 R\sqrt{-g}$ is important to the current experimental tests of Newton's law [15] that does not contradict with coupling constants a and b have value: $a \approx b \sim 10^{60}$. The current experimental constraints from sub-millimeter tests to corrections of higher-curvature terms [16,17] to the Newtonian potential, give for dimensionless constants a and b bounding $a,\, b < 10^{62}$.

The field equations for metric $g_{\mu\sigma}$ and the Higgs field Φ following from the action (1) have solutions $g^{(0)}_{\mu\sigma} = \eta_{\mu\sigma}$ is the Minkowski metric as the metrical ground state and nontrivial Higgs field ground state is $\left(\Phi^+\Phi\right)_0 = v^2 \approx (10^3 Gev)^2$.

The standard way in perturbative theory one writes the metric as $g_{\mu\sigma} = \eta_{\mu\sigma} + \tilde{h}_{\mu\sigma}$. In the unitarity gauge Higgs field takes the form, avoiding Goldstone bosons, we set

$$\Phi = \begin{pmatrix} 0 \\ v + \varphi \end{pmatrix} \quad , \tag{2}$$

where the real scalar field $\varphi(x)$ describes the excited Higgs field connected with the Higgs particle.

The part of the action (1) quadratic in the fields $\tilde{h}_{\mu\sigma}$ and φ can be written as

$$S = \int d^4x[\epsilon v\varphi R^{(1)}(\widetilde{h}) + aW(\widetilde{h}) - \frac{b}{3}(R^{(1)}(\widetilde{h}))^2 - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{8fv^2}{2}\varphi^2], \tag{3}$$

where the Ricci tensor $R^{(1)}_{\mu\sigma}(\tilde{h})$ and the scalar curvature $R(\tilde{h})$ can be written in a linearized form

$$R^{(1)}_{\mu\rho}(\tilde{h}) = \frac{1}{2}(\Box\tilde{h}_{\mu\rho} - \partial_\mu\partial_\sigma\tilde{h}^\sigma_\rho - \partial_\rho\partial_\sigma\tilde{h}^\sigma_\mu + \partial_\mu\partial_\rho\tilde{h}), \tag{4}$$

$$R^{(1)}(\tilde{h}) = (\Box\tilde{h} - \partial^\rho\partial^\sigma\tilde{h}_{\rho\sigma}) \qquad (5)$$

and $W(\tilde{h})$ is:

$$W(\tilde{h}) = (R^{(1)}_{\mu\rho}(\tilde{h}))^2 - \frac{1}{3}(R^{(1)}(\tilde{h}))^2. \qquad (6)$$

In the expression (3) for the fields $\tilde{h}_{\mu\rho}$ and φ has the unwanted mixed term :

$$\epsilon v\varphi R^{(1)}(\tilde{h}).$$

We can be rid of this term making the following redefined field

$$\tilde{h}_{\mu\rho} = h_{\mu\rho} + \frac{\eta_{\mu\rho}v}{4\epsilon}\Box^{-1}\varphi. \qquad (7)$$

We find that the terms $R^{(1)}(\tilde{h})$ and $W(\tilde{h})$ take the forms :

$$R^{(1)}(\tilde{h}) = R^{(1)}(h) + \frac{3v}{4\epsilon}\varphi \qquad (8)$$

and

$$W(\tilde{h}) = W(h) \qquad (9)$$

we will not keep total derivative term in eq.(9).

Putting expressions (8) and (9) in the action (3) we get the following condition rid of the mixed term

$$\epsilon^2 = \frac{b}{2} \qquad (10)$$

for gravitational constants ϵ and b.

Thus, the Planck mass M_p is not the fundamental scale and depends on the coupling constant b by quadratic curvature term and the electroweak scale $v \approx 10^3 Gev$ which is the fundamental scale:

$$M_p = (\frac{b}{2})^{\frac{1}{4}}v \approx 10^{15}10^3 Gev \sim 10^{18} Gev. \qquad (11)$$

As a result, expression (3) has the following form

$$S = \int d^4x[aW(h) - \frac{b}{3}(R^{(1)}(h))^2 - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}(8f - \frac{3}{4})v^2\varphi^2], \qquad (12)$$

where $(8f - \frac{3}{4})v^2 = m^2_\varphi$ is square mass of the Higgs particle at $(8f - \frac{3}{4}) \geqslant 0$.

Of course, the differential operator which appears in the gravity part of action (12) is not invertible. It is necessary to add a gauge- fixing term, in case

$$S_{GF} = -\frac{1}{2\alpha}\int(\partial^\sigma h_{\sigma\mu}\eta^{\mu\lambda}\Box\partial^\rho h_{\rho\lambda})d^4x. \tag{13}$$

Going over to momentum space and using the projectors[1,7,18] for the spin-two $P^{(2)}_{\mu\rho\lambda\sigma}$, spin-one $P^{(1)}_{\mu\rho\lambda\sigma}$ and the two spin-zero $P^{(0-s)}_{\mu\rho\lambda\sigma}$ and $P^{(0-w)}_{\mu\rho\lambda\sigma}$ we find for actions (12) and (13)

$$\widetilde{S} = S + S_{GF} = \frac{1}{2}\int h^{\mu\rho}\{k^4[\frac{a}{2}P^{(2)} + \frac{1}{2\alpha}P^{(1)} - 2bP^{(0-s)}+ \tag{14}$$

$$+\frac{1}{\alpha}P^{(0-w)}]_{\mu\rho\lambda\sigma}\}h^{\lambda\sigma}d^4k.$$

Then the propagator for the fields $h_{\lambda\rho}$ in the momentum space is

$$D_{\mu\rho\lambda\sigma} = \frac{2}{ak^4}P^{(2)}_{\mu\rho\lambda\sigma} + \frac{2\alpha}{k^4}P^{(1)}_{\mu\rho\lambda\sigma} - \frac{1}{2bk^4}P^{(0-s)}_{\mu\rho\lambda\sigma} + \frac{\alpha}{k^4}P^{(0-w)}_{\mu\rho\lambda\sigma} \tag{15}$$

the components projectors by $P^{(1)}$ and $P^{(0-w)}$ can be gauged away at $\alpha \longrightarrow 0$.

Ignoring the terms proportional α, we have for the propagator of the momentum space is

$$D_{\mu\rho\lambda\sigma} = \frac{2}{ak^4}P^{(2)}_{\mu\rho\lambda\sigma} - \frac{1}{2bk^4}P^{(0-s)}_{\mu\rho\lambda\sigma}.$$

In this letter we do not discuss the possible running of the coupling constant a, b,and f with increasing momentum.

Let us note that redefinition (7) brings the contribution $\frac{v}{\epsilon k^2}$ to some vertex of the Feynman diagrams.

In this case tree-level mass of the Higgs particle is zero (which the coupling constant f is $f = \frac{3}{32}$). We only have quantum gravitational corrections in the mass of Higgs particle and the vacuum energy.

According to quantum corrections we have the following form of the vacuum energy density

$$\rho_{vac} \simeq \frac{3}{4b}v^4 < 10^{-48}Gev^4 \tag{16}$$

at $a \approx b \sim 10^{60}$. Thus in the framework new version R^2-gravity with one scale, which is the electroweak scale $v \approx 10^3 Gev$ can be also find of the solution smallness problem of the cosmological constant.

Vacuum energy density (16) is an example of the dark energy, which counts to about 75 per cent of the total energy density. As a result, the universe expansion is acceleration [19].

References

1. K.S. Stelle, Phys. Rev. **D16,** 953 (1977).

2. J. Julve, M. Tonin, Nuovo Cim. **B46,** 137 (1978).
3. A. Salam, J. Strathdee, Phys. Rev. **D18,** 4480 (1978).
4. I.G. Avramidi, A.D. Barvinsky, Phys. Lett. **B159,** 269 (1985).
5. E. Fradkin, A.A.Tseytlin, Nucl. Phys. **B201,** 469 (1982);
6. A. Codello, R. Percacci, Phys. Rev. Lett. **97,** 221301 (2006).
7. I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro,
   Effective action in quantum gravity,
   IOP Publishing (Bristol and Philadelphia, 1992).
8. S. Adler, Rev. Mod. Phys. **54,** 729 (1982).
9. A. Zee, Phys. Rev. **D23,** 858 (1981).
10. A. Zee, Ann. Phys. **151,** 431 (1983).
11. B.Hasslasher, F. Mottola, Phys. Lett. **95B,** 237 (1980).
12. I. L. Buchbinder, S. D. Odintsov, Class. Quantum Grav. **2,** 721 (1985).
13. F. David, A. Strominger, Phys. Lett. **143B,** 125 (1984).
14. S. Weinberg, Phys. Rev. Lett. **19,** 1264 (1967).
15. D. J. Kapner, T. S Cook, E.G. Adelberger, J. H.Gundlalach,
    B. R. Heckel, C. D. Joyle, H. E. Swanson,
    Phys. Rev.Lett. **98,** 021101 (2007);
16. J.F. Donoghue, Phys. Rev. **D50**, 3874 (1994);
17. K.S. Stelle, Gen. Rel. Grav. **9,** 353 (1978).
18. P. van Nieuwenhuizen, Nucl. Phys. **B60,** 478 (1973).
19. A. G. Riess, et al., Astrophysics J. **607,** 665 (2004).